

RELATIVISTIC EFFECTS OF MIXED VECTOR-SCALAR-PSEUDOSCALAR POTENTIALS FOR FERMIONS IN 1+1 DIMENSIONS

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Abstract

The problem of fermions in the presence of a pseudoscalar plus a mixing of vector and scalar potentials which have equal or opposite signs is investigated. We explore all the possible signs of the potentials and discuss their bound-state solutions for fermions and antifermions. The cases of mixed vector and scalar Pöschl-Teller-like and pseudoscalar kink-like potentials, already analyzed in previous works, are obtained as particular cases.

1 Introduction

The pseudospin symmetry has been used to explain a number of phenomena in nuclear structure.[1] In order to understand the origin of pseudospin symmetry, we need to take into account the motion of the nucleons in a relativistic mean field and thus consider the Dirac equation.[2] The cases in which the mean field is composed by a vector (V_t) and a scalar (V_s) potential, with $V_t = V_s$ ($V_t = -V_s$), are usually pointed out as a necessary condition for occurrence of pseudospin (spin) symmetry in nuclei.[2] Furthermore, with an appropriate choice of signs, potentials fulfilling the relations $V_s = \pm V_t$ are able to bind either fermions or antifermions.[3] Moreover, in the nucleus, the charge-conjugation transformation relates the spin symmetry of the negative bound-state solutions (antinucleons) to the pseudospin symmetry of the positive bound-state solutions (nucleons).[4] In addition, a link of the pseudospin to chiral symmetry has been suggested.[5] The main motivation of this article is to shed some light in the relation between spin and pseudospin symmetries by means of charge-conjugation and chiral transformations in 1+1 dimensions. To do so, we solve the mixed scalar-vector-pseudoscalar Pöschl-Teller potential in 1+1 dimensions, taking advantage of the simplicity of the lowest dimensionality of the space-time as much as was done for the harmonic oscillator.[6]

2 The Dirac equation in a 1+1 dimensions

The 1+1 dimensional time-independent Dirac equation for a fermion of rest mass m under the action of vector (V_t), scalar (V_s) and pseudoscalar (V_p) potentials can be written, in terms of the combinations $\Sigma = V_t + V_s$ and $\Delta = V_t - V_s$, as

$$H\Psi = E\Psi, \quad H = c\sigma_1 p + \sigma_3 mc^2 + \frac{1 + \sigma_3}{2}\Sigma + \frac{1 - \sigma_3}{2}\Delta + \sigma_2 V_p, \quad (1)$$

where σ_1, σ_2 and σ_3 denote the Pauli matrices.

The *charge-conjugation operation* is accomplished by the transformation $\Psi_c = \sigma_1 \Psi^*$ and the Dirac equation becomes $H_c \Psi_c = -E \Psi_c$, with

$$H_c = c\sigma_1 p + \sigma_3 mc^2 - \frac{1 + \sigma_3}{2}\Delta - \frac{1 - \sigma_3}{2}\Sigma - \sigma_2 V_p. \quad (2)$$

The chiral operator for a Dirac spinor is the matrix $\gamma^5 = \sigma_1$. Under the *discrete chiral transformation* the spinor is transformed as $\Psi_\chi = \gamma^5 \Psi$ and the transformed Hamiltonian $H_\chi = \gamma^5 H \gamma^5$ is

$$H_\chi = c\sigma_1 p - \sigma_3 mc^2 + \frac{1 + \sigma_3}{2} \Delta + \frac{1 - \sigma_3}{2} \Sigma - \sigma_2 V_p. \quad (3)$$

We have two first-order equations for the upper, ψ_+ , and the lower, ψ_- , components of the spinor:

$$-i\hbar c\psi'_- + mc^2\psi_+ + \Sigma\psi_+ - iV_p\psi_- = E\psi_+ \quad (4)$$

$$-i\hbar c\psi'_+ - mc^2\psi_- + \Delta\psi_- + iV_p\psi_+ = E\psi_-, \quad (5)$$

where the prime denotes differentiation with respect to x .

For $\Delta = 0$ and $E \neq -mc^2$, the Dirac equation becomes

$$\psi_- = -i(\hbar c\psi'_+ - V_p\psi_+)/ (E + mc^2), \quad (6)$$

$$-\hbar^2 c^2 \psi''_+ + [(E + mc^2)\Sigma + V_p^2 + \hbar c V'_p] \psi_+ = (E^2 - m^2 c^4) \psi_+, \quad (7)$$

The chiral symmetry is invoked to obtain the equations obeyed by ψ_+ and ψ_- , for $\Sigma = 0$ and $E \neq mc^2$. They are obtained from the previous ones by doing $\psi_+ \leftrightarrow \psi_-$, $m \rightarrow -m$, $\Sigma \rightarrow \Delta$ and $V_p \rightarrow -V_p$. Thus, either for $\Delta = 0$ with $E \neq -mc^2$ or $\Sigma = 0$ with $E \neq mc^2$ the solution of the relativistic problem is mapped into a Sturm-Liouville problem in such a way that solutions can be found by solving a Schrödinger-like problem. The solutions for $\Delta = 0$ with $E = -mc^2$ and $\Sigma = 0$ with $E = mc^2$, excluded from the Sturm-Liouville problem, are obtained directly from the original first-order equations.

3 The Pöschl-Teller-like potential

Let us consider

$$\Sigma = -\hbar c|\alpha|g_1 \operatorname{sech}^2 \alpha x, \quad \Delta = 0, \quad V_p = \hbar c|\alpha|g_2 \tanh \alpha x. \quad (8)$$

Due to the chiral symmetry one can restrict the discussion to the Σ case ($\Delta = 0$). The results for the case when $\Delta = -\hbar c|\alpha|g_1 \operatorname{sech}^2 \alpha x$, $\Sigma = 0$, $V_p = \hbar c|\alpha|g_2 \tanh \alpha x$ are obtained by changing the sign of m and of g_2 in the relevant expressions. The chiral symmetry can also be seen in the isolated solutions, which are converted into each other by this transformation.

For the potential in (8) we have not found a normalizable isolated solution with $E = -mc^2$ and for $E \neq -mc^2$, Eq. (7) takes the form

$$-\frac{\hbar^2}{2m_{\text{eff}}} \psi_+'' - U_0 \text{sech}^2 \alpha x \psi_+ = E_{\text{eff}} \psi_+, \quad (9)$$

where

$$U_0 = (\hbar c \alpha)^2 [g_2(g_2 - 1) + (g_1/\hbar c |\alpha|)(E + mc^2)] / 2m_{\text{eff}}, \quad (10)$$

$$E_{\text{eff}} = (E^2 - m_{\text{eff}}^2 c^4) / 2m_{\text{eff}}, \quad m_{\text{eff}} = \sqrt{m^2 + (\hbar \alpha g_2 / c)^2} \quad (11)$$

The well-behaved solutions for Eq. (9), with U_0 necessarily real and positive, are the solutions of the Schrödinger equation for the nonrelativistic symmetric modified Pöschl-Teller potential.[7] Following these previous references and using Eqs. (10) and (11) we get the quantization condition for the energy:

$$\hbar c |\alpha| \sqrt{1 + 4 \left[g_2(g_2 - 1) + \frac{g_1}{\hbar c |\alpha|} (E + mc^2) \right]} - 2\sqrt{m_{\text{eff}}^2 c^4 - E^2} = \hbar c |\alpha| (2n + 1). \quad (12)$$

In general there is no requirements on the signs of g_1 and g_2 , except that $U_0 > 0$. From Eq. (10), the Dirac eigenenergies corresponding to the bound-states solutions must be within the limits

$$E \gtrless -mc^2 \mp \frac{g_2(g_2 - 1)}{g_1} \hbar c |\alpha|, \quad \text{for } g_1 \gtrless 0 \quad (13)$$

For $g_1 = 0$, we obtain the pseudoscalar kink-like potential.[8] When $g_2 = 0$ we obtain the mixed vector-scalar Pöschl-Teller-like potential with spin symmetry.[9] Eq. (12) was solved with a symbolic algebra program by searching eigenenergies in the ranges in (13). Fig. 1 (left) shows the behavior of the energies for $g_1 = -10$, for several values of g_2 . For $g_1 = 10$ the spectrum presents similar behavior if we reverse the vertical axis. Fig. 1 (right) illustrates the behavior of the energies as a function of g_1 for $g_2 = 10$.

The case $\Sigma = 0$ can be obtained from the $\Delta = 0$ case by applying the charge-conjugation transformation. We recall that this transformation performs the changes $E \rightarrow -E$, $\Delta \leftrightarrow -\Sigma$, $V_p \rightarrow -V_p$, or, the changes $g_1 \rightarrow -g_1$ and $g_2 \rightarrow -g_2$. This can be seen if we solve the eigenvalue equation for $\Sigma = 0$, for $g_1 = 10$ and plot the solutions for several values of

g_2 . If we compare that figure with Fig. 1 (left) ($g_1 = -10$) we see that the plots are identical if we reverse both the vertical and horizontal axes, i.e, if we reverse the sign of the energy and of g_2 concomitantly. Of course, the identification of particle and antiparticle levels is also reversed, as it should be, because we are applying the charge-conjugation transformation. All of that implies into the possibility of bounded states with both positive- and negative-energy solutions in a system with $\Sigma = 0$, that can be relevant for nuclei.

Finally we stress that the chiral transformation enables us to switch between the so-called pseudospin ($\Sigma = 0$ and $V_p = 0$) and spin symmetries ($\Delta = 0$ and $V_p = 0$) *for any potential* and, for massless fermions, it shows that the $\Delta = 0$ and $\Sigma = 0$ solutions have the same spectrum. These conclusions seem to remain true in 3+1 dimensions, because the 1+1 eigenvalue equation for $\Delta = 0$ and $\Sigma = 0$ are very similar to the corresponding 3+1 equations.

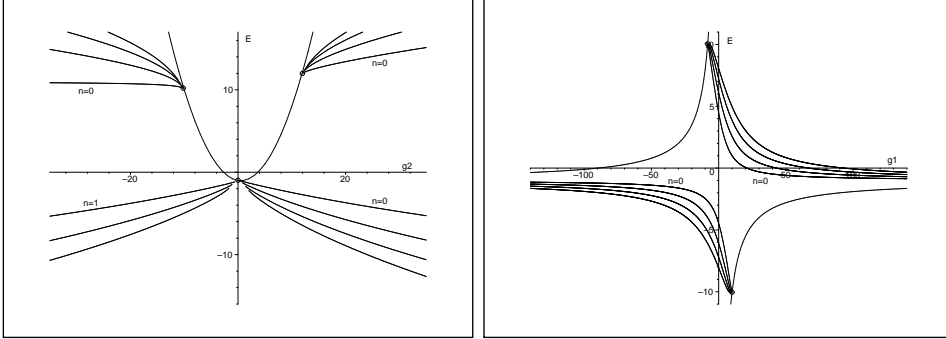


Figure 1: (left) Dirac eigenvalues for the four lowest energy levels when $g_1 = -10$ as a function of g_2 . (right) Dirac eigenvalues for the four lowest energy levels when $g_2 = 10$ as a function of g_1 . The thin line represents the function $-(1 + \frac{g_2}{g_1})(g_2 - 1)$ ($m = \hbar = c = \alpha = 1$).

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